Protection for Sale: The Case of Oligopolistic Competition and Interdependent Sectors[*]

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Abstract

In Grossman and Helpman's (1994) canonical "Protection for Sale" (PFS) model political competition between industry lobbies is purely driven by their interests as consumers. This paper introduces demand linkages and oligopolistic competition into PFS framework to address the rivalry among lobbies due to product substitutability. It shows that increased substitutability weakens the interest groups' incentives to lobby and reduces tariff distortions. This may explain why empirical tests of PFS find surprisingly little impact of lobbies on the government trade policy decision. The paper also analyzes endogenous lobby formation, suggesting that demand linkages may adversely affect industry decision to get organized.

Keywords: Lobbying, Endogenous Trade Policy, Substitutability

JEL codes: F12, F13, D72

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1 Introduction

This paper studies the effect of demand linkages on lobbying for trade policy. It introduces product substitutability and oligopolistic market structure into the canonical Grossman and Helpman's (1994) "Protection for Sale" framework (henceforth PFS) to analyze how the political competition among the industrial lobbies is affected by interdependency of their demands. It shows that increased product substitutability may weaken both the lobbying intensity and the incentives of the industries to get organized. Both effects reduce the lobbying-driven tariff distortions, which may explain why the empirical tests of PFS commonly estimate the government to be surprisingly benevolent in its trade policy decisions.

While there is a sizable theoretical literature addressing the impact of the organized lobby groups on trade policy (see e.g. Rodrik (1995) for a review), PFS model is the most influential by far. PFS explicitly describes a mechanism through which interest groups' contributions influence the policy-maker's decision for trade protection, providing micro-foundations for the previous approaches. In addition, the model's prediction for the equilibrium protection pattern relates the industry's trade tariff to a number of observable variables, thereby providing a coherent framework for empirical testing.

As is well known, PFS neglects some important issues. In particular, it abstract from strategic market interactions and production linkages. Indeed, the demand structure in PFS implies that lobby groups do not behave strategically in the product markets. Also, factor-specific production eliminates any competition between different lobbies in the factor market. Hence, their interests only diverge to the extent that each industry wants to increase its profits by raising the price of its own good, while all other organized industries aim at reducing the same price in order to increase their members' consumer surplus. That is, political rivalry among the lobbies arises purely from the desire of the members of different industry lobbies to defend their interests as consumers, as is recognized by Grossman and Helpman (p.849). [1]

This paper adds a more realistic justification for the political competition among the organized groups by considering strategic interactions between the industries due to their products' substitutability. It studies the impact of these demand-driven interactions on the determination of trade policy, the intensity of inter-industry lobbying competition and lobby formation.

In the original PFS model the demand for each good is independent of the prices of other goods. To address the demand-driven rivalry in lobbying we relax this assumption and allow for product substitutability. However, even with substitutability, the small competitive economy setting of PFS excludes any demand-side strategic interactions between industries. Indeed, in PFS each good produced from

[1] See e.g. Baldwin and Robert-Nicoud (2006) for further discussion of concerns caused by this feature of PFS.

labor and a sector-specific factor in an inelastic supply, so each industry has a positively sloped supply curve. For a given (non-prohibitive) tariff, a demand shift due to substitutability would only impact imports, but not domestic output or profits. So, to study demand-driven lobby behavior, we also introduce imperfect competition, assuming that each good is produced by an international oligopoly and sold in internationally segmented markets. We show that the oligopolistic markets *per se* do not alter key PFS predictions - e.g., that the organized industries are overprotected, and the non-organized industries underprotected. This allows us to concentrate on the effects of product substitutability.

The first part of the paper shows that the presence of substitutes may reduce interest group incentives to lobby due to competition in the goods market. If demands are interdependent, an increase in the price of a good causes its demand to shift towards the substitutes. A decrease in the price of the substitute has a similar effect. The interest group takes this into account when lobbying the government to increase the price of its own good (as a producer receiving profit from selling this good) and reduce the price of all other goods (as a group of consumers). Therefore, the lobbying strategy of an organized industry becomes less aggressive. As a result, with an increase in the degree of substitutability, the protection of the organized industries falls, and the protection of the non-organized industries increases, relative to the first-best benchmark. That is, other things equal, the trade tariffs in economies producing more substitutable products should be closer to the socially optimal levels.

This result suggests a new explanation for the known puzzle of the empirical studies of the PFS model. These studies commonly find that the government puts unexpectedly low weight on the lobby contributions relative to the welfare loss.[2] That is, the interest groups are found to have surprisingly little impact on the government trade policy decision, which causes a concern about the empirical significance of the PFS model. E.g., Gawande and Krishna (2003) write that "..it is enough to cast doubt on the value of viewing trade policy determination through this political economy lens".

Existing explanations for this finding can be roughly classified in two groups. The first group argues that empirical tests of PFS model neglect the presence of other interest groups with objectives opposed to those of organized groups in PFS. The conflicting objectives imply that the lobbying efforts of interest groups would offset each other, leading to less protection. For example, Gawande and Bandyopadhyay (2000) introduce political competition between the upstream and downstream producers, and Gawande et. al. (2006) study foreign lobbies. The second group attributes the puzzling finding to overestimated substitutability between domestic and foreign goods within the same industry. Facchini et. al. (2010) argue that, if the domestic and foreign goods are imperfect substitutes, domestic organized groups would

[2] E.g. Goldberg and Maggi (1999), Gawande and Bandyopadhyay (2000) or Mitra et al. (2002)

be less interested in protection, which leads to lower trade barriers. This paper provides an alternative explanation, suggesting that smaller deviations from the first-best protection rates may result from product substitutability between the industries, which is neglected by the empirical tests of PFS.

The second part of the paper addresses the impact of product substitutability on the incentives to form a lobby. The original PFS model assumes an exogenous lobby group structure. A natural extension is to endogenize the lobby formation. However, in PFS the interests of different industry groups are opposed to each other. So if an additional lobby formation stage is introduced in their model, all industries would likely get organized (in the absence of lobby formation costs).[3]

However, in the presence of demand linkages, at a sufficiently high degree of substitution a non-organized industry becomes protected without paying for it. The reason is that the organized industry cannot lobby to increase its own price (or lower the price of the substitute) without losing part of its consumers switching to the cheaper (but similar) good. Thus, the non-lobbying industry gets a free ride on the lobbying industry efforts. If instead both these substitute-producing industries are organized, they both contribute to the government for (potentially higher) protection. Comparing these two outcomes, we demonstrate that the industry may better off not being organized. As a result, with endogenous lobby formation, fewer industries get organized and lobbying becomes less intense.

There are several studies that extend the PFS framework to address the political rivalry among the organized groups. Gawande and Bandyopadhyay (2000) introduce supply-side interactions through a single importable intermediate input. Cadot et. al. (2004) assume that the industries compete for a common scarce resource and each good is produced using other goods as intermediate inputs. However, to our knowledge, this paper is the first one to study the rivalry arising from the demand-side interactions.[4] A related paper, Bombardini and Trebbi (2012), considers oligopolistic markets with differentiated products and discusses the lobbying incentives arising from product substitutability. Yet, their setting differs from the PFS approach. In particular, in their paper the organized groups can only lobby either for the own tariff or for economy-wide one. This restriction limits the possibility of addressing the political competition among the lobbies. In addition, their model relies on Bertrand competition, implying prohibitive tariffs and zero imports of differentiated goods. While the logic of our modelling approach works also in the case of price competition, most of our results are based on Cournot competition, thereby avoiding such an extreme prediction.

This paper is not the first to endogenize lobby formation in the PFS model or discuss the possibility

[3] Similarly to PFS, we neglect any intra-industry organizational conflict.

[4] Chang (2005) incorporates monopolistic competition into the PFS framework, but in her setup there is substitutabillity only across varieties within the same sector, and no demand linkages across sectors.

of free-riding. Mitra (1999) adds an initial stage to the PFS model, when industries can get organized at a cost. In Magee (2002), in the first stage, industry representatives and the policy maker determine the tariff schedule and, in the second stage, every firm in the industry decides whether to contribute to the lobbying effort (defecting is infinitely punished). Bombardini (2008) looks at the lobbying incentives of individual firms, again assuming a fixed cost of lobby participation. These papers address the collective action problem at the intra-industry level. We, instead, relate lobby formation to the inter-industry demand links, thereby avoiding the issue of exogenous fixed costs. As mentioned above, Bombardini and Trebbi (2012) also consider the impact of inter-industry links on lobbying organization. However, they study the incentives for joint vs. individual lobbying rather then endogenous lobby formation.

The remainder of the paper is organized as follows: Section 2 describes the model setup, Section 3 discusses the equilibrium structure of protection in the presence of demand linkages, Section 4 analyzes the effect of the degree of substitution on the extent of protection, Section 5 studies endogenous lobby formation and Section 6 concludes. All proofs are relegated to the appendix.

2 The model

Consider an economy populated by individuals of total mass 1. They have identical preferences represented by the quasi-linear utility function

$$U(x_0, x_1, ..., x_m) = x_0 + \hat{U}(x_1, ..., x_m), \tag{1}$$

where x_i denotes consumption of good i, there are $m+1$ goods in the economy, and good 0 is chosen to be a numeraire. We assume quadratic sub-utility function

$$\hat{U}(x_1, ..., x_m) = \sum_{k=1}^{m} x_k - 1/2 \sum_{k=1}^{m} x_k^2 - \sigma \sum_{k=1}^{m} \sum_{j=k+1}^{m} x_k x_j, \tag{2}$$

where $\sigma \in [0, 1]$ reflects the substitutability between goods $x_1, ..., x_m$.

Functional form (2) yields a familiar linear inverse demand function for goods $i = 1, ..., m$

$$p_i = 1 - x_i - \sigma \sum_{j \neq i}^{m} x_j,$$

where p_i denotes the domestic price of good i, and $\mathbf{p} = (p_1, ..., p_m)$ is the vector of domestic prices. For $\sigma \in (0, 1)$, considering only interior solutions, the resulting demand function for good i is linear as well

$$d_i(\mathbf{p}) = \frac{1}{((m-1)\sigma + 1)(1-\sigma)} \left((1-\sigma) - (1 + (m-2)\sigma)\, p_i + \sigma \sum_{j \neq i}^{m} p_j \right). \tag{3}$$

The demand for good i decreases in its own price and increases in prices of all other non-numeraire goods, so they are (imperfect) substitutes. These cross-price effects constitute the main difference with the original PFS model.[5] Due to the quasi-linearity of the utility function, any income effect is totally captured by the consumption of good 0.

There is a single factor of production, labor. Good 0 is produced with an input-output coefficient of 1 and freely traded in a perfectly competitive international market. As a result, the domestic wage rate is equal to 1. The other m goods are sold at internationally segmented oligopolistic markets. More precisely, good k is supplied by $n_k > 0$ identical domestic firms and $n_k^* > 0$ identical foreign firms, which compete in quantities. The total number of firms in sector k is given by $N_k = n_k + n_k^*$. It takes c_k units of labor to produce one unit of good k, both at home and abroad. Each firm is a profit maximizer and the number of firms is fixed, so no entry or exit is considered.

The government in the economy sets trade taxes and/or subsidies on the goods $i = 1, ..., m$. Resulting net revenue is redistributed equally among the population.

Due to the segmentation of the domestic and foreign markets and CRS technology, firms' production decisions and government trade policy decisions can be made separately for the domestic and the international market. We focus on the government decision about import tariffs, which affect domestic market. We also abstract from strategic trade policy interaction between the domestic and foreign governments assuming that the foreign government does not impose any export tariffs on its firms.

Denote the domestic import tariff in sector k by τ_k, and the vector of the import tariffs $(\tau_1, ..., \tau_m)$ by τ. The Cournot-Nash equilibrium output of each domestic firm in sector k is denoted by $q_k(\tau)$, and of each foreign firm - by $q_k^*(\tau)$. The profits are given by $\pi_k(\tau)$ and $\pi_k^*(\tau)$ for a domestic and foreign firm, respectively.

Each individual is endowed with some labor and may also own (indivisible and non-tradable) claims to the profit of a firm in at most one industry. Thus, individual income is the sum of wages, government transfer and possibly claims to a domestic firm's profit. The owners of firms in the same industry i may choose to organize in a lobby group trying to influence government's trade policy decision. The joint welfare of the members of such a group comprising share α_i of the population is

$$W_i(\tau) = l_i + n_i \pi_i(\tau) + \alpha_i \left[\sum_{k=1}^{m} \tau_k m_k(\tau) - \sum_{k=1}^{m} p_k d_k(\mathbf{p}(\tau)) + \hat{U}(d_1(\mathbf{p}(\tau)), ..., d_m(\mathbf{p}(\tau))) \right], \qquad (4)$$

where l_i denotes the total labor endowment of group i, $m_k(\tau) = n_k^* q_k^*(\tau)$, is the import in sector k, and $\sum_{k=1}^{m} \tau_k m_k(\tau)$ is the tariff revenue. Lobby i's contribution to the government $C_i(\tau)$ is conditioned on the trade policy vector.

[5] The original PFS model uses the separable utility function $U(x_0, x_1, ..., x_m) = x_0 + \sum_{i=1}^{m} \hat{u}_i(x_1, ..., x_m)$, which implies that demand functions are independent of the prices of the other goods.

The objective function of the government is

$$G(\tau) = \sum_{i \in L} C_i(\tau) + aW(\tau),$$

where L is the exogenously given set of organized sectors, $W(\tau) = \sum_{k=1}^{m} W_i(\tau)$ is the aggregate welfare in the economy, and $a > 0$ is the weight the government attaches to aggregate welfare.

The game timing is as follows: first, lobbies simultaneously announce their contribution schedules, i.e., the amount contributed as a function of the import tariffs vector, Then the government chooses trade policy. Finally firms make their production decisions and consumption takes place.

3 Protection and substitutability

In this section we characterize an equilibrium of the game and compare the resulting trade policy with and without substitutability between the goods.

Similarly to PFS approach, we focus on locally truthful equilibria of this menu-auction game, i.e., equilibria where the marginal change in each lobby's contribution to the government resulting from a small policy change is exactly equal to the marginal change of this lobby's welfare. This yields the following condition (equation (12) in PFS)

$$\sum_{i \in L} \nabla W_i(\tau) + a \nabla W(\tau) = \mathbf{0}. \tag{5}$$

Using the expressions for the lobby's welfare (4) and aggregate welfare, system (5) can be rewritten as

$$\tau_i = \sum_{j=1}^{m} h_{ij} \left(\sum_{k=1}^{m} \frac{(I_k + a)}{(a + \alpha_L)} n_k \frac{\partial \pi_k(\tau)}{\partial \tau_j} - \sum_{k=1}^{m} \frac{\partial p_k}{\partial \tau_j} d_k(\mathbf{p}(\tau)) + n_j^* q_j^*(\tau) \right), \quad i = 1, ..., m, \tag{6}$$

where I_k equals 1 if industry k is organized, and 0 otherwise, $\alpha_L = \sum_{i \in L} \alpha_i$ is the total share of population in the organized industries, and h_{ij} are the elements of matrix $H = -\left[\left(\partial m_k(\tau)/\partial \tau_j \right)_{k,j} \right]^{-1}$ - the negative inverse of the matrix of the derivatives of imports with respect to the trade tariff. System (6) characterizes the equilibrium trade policy and in what follows we analyze this system.[6]

First, we show that the oligopolistic competition *per se* does not alter the key PFS predictions - that the organizes industries are overprotected, and the non-organized ones underprotected. To do so we neglect any substitutability in demand, setting $\sigma = 0$. With all cross-derivatives being zero system (6) turns into a set of independent equations

$$\tau_i = \frac{1}{(-\partial m_i(\tau)/\partial \tau_i)} \left[\frac{(I_i + a)}{(a + \alpha_L)} n_i \frac{\partial \pi_i(\tau)}{\partial \tau_i} - \left(\frac{\partial p_i}{\partial \tau_i} d_i(\mathbf{p}(\tau)) - n_i^* q_i^*(\tau) \right) \right]. \tag{7}$$

[6] Local truthfulness requires differentiability of contribution schedules (and imports, as a result). Thus, here and thereafter we only consider interior solutions (i.e, non-prohibitive tariffs).

which yield an easy solution for the equilibrium tariff rate

$$\tau_i = (1 - c_i) \bigg/ \left(\frac{(n_i + 1)(N_i + 1)}{\left(\frac{I_i + a}{a + \alpha_L} n_i + \frac{1}{2} \right)} - n_i^* \right). \tag{8}$$

In turn, the first-best tariff in absence of any lobbying τ_i^0 is given by expression (8) with $\alpha_L = I_i = 0$, so that the term $(I_i + a) / (a + \alpha_L) = 1$.

Due to imperfect competition in the goods markets, free trade is no longer socially optimal. But, just as in the original PFS model, the organized industries (those with $I_i = 1$) experience higher protection than in the first-best equilibrium, as long as not everyone belongs to a lobby group ($\alpha_L < 1$). Indeed for these industries $(I_i + a) / (a + \alpha_L) > 1$, so $\tau_i > \tau_i^0$. The reverse is true for the non-organized sectors.[7]

Now, we are ready to study the equilibrium import tariffs in presence of product substitutability by analyzing system (6) with $\sigma > 0$. Unlike the case with a separable utility (system (7)), now trade tariffs are directly affected by the sensitivity of supply and demand in the other industries and the organizational status of these industries.

In particular, the negative effect of industry l being organized on industry i's protection is weaker if industries i and l produce substitutes. To see this, assume that industry l is getting organized. In case of independent demands (system (7)) this reduces the protection for industry i only through an increase in α_L (as domestic profits increase in the own tariff, $\partial\pi_i(\tau)/\partial\tau_i > 0$, see Lemma A1 in the Appendix). With substitute goods, other things equal, industry l getting organized reduces the industry i tariff through higher α_L in factors $(I_k + a)/(a + \alpha_L)$ for all $k \in 1, ..., m$ (again, Lemma A1 shows that $\partial\pi_k(\tau)/\partial\tau_j \geq 0$ and $h_{ij} \geq 0$ for all i, j). However, it also has a positive effect on industry i's protection through an increase in I_l from 0 to 1 in factor $(I_l + a)/(a + \alpha_L)$.

The intuition behind this effect is straightforward: In absence of substitution each organized group attempts to increase the trade tariff on its own good to raise the profit, and to reduce the tariffs on all other goods to lower the prices of its consumption bundle. However, in the presence of substitutability between the goods, such a lobbying strategy may cause a demand loss, as consumers would switch from more protected (and thus, more expensive) goods to less protected ones. To limit substitution, organized industries tend to apply more "moderate" lobbying strategies. That is, they try to maintain a balance between decreasing the price of the other goods and increasing its own price.

This effect is best understood in case the ownership of industry l is highly concentrated, $\alpha_l = 0$. Then, the effect of other industries' prices on lobby l's consumer welfare is negligible. Therefore, in the ab-

[7] If all industries are organized and every voter belongs to some lobby ($\alpha_L = I_i = 1\ \forall j$), the protection rates are equal to the first-best ones, exactly like in PFS.

sence of substitutability, if industry l gets organized, it does not affect the protection of any other industry $i \neq l$ (indeed, as the total share of the organized population α_L does not change, tariff equations (7) for $i \neq l$ are unaffected). However, with substitutability, industry l still lobbies for additional protection of all substitute-producing industries because it is concerned with maintaining its own demand.

In PFS, the interests of different industry groups are opposed to each other. Here, we see that non-organized industries may benefit from the contributions of organized ones by "exploiting" the demand properties. We return to this discussion in the subsequent sections and show that this effect may lead to free-riding in the lobbying behavior.

The above results are obtained for Cournot competition. However, it is easily seen that they also hold if we allow firms to compete in prices in Bertrand-fashion. If domestic and foreign goods are homogenous, the only interesting case to consider is the one of a single domestic firm. Otherwise, Bertrand competition would drive domestic firms' profits to zero independently of the tariffs, so there will be no lobbying. In domestic monopoly case, positive tariff imply that the market is served by the domestic firm at the price of $p_i = c_i + \tau_i$. Under negative tariff all sales will be done by the foreign producers at the price $p_i \leq c_i$. In the absence of substitutability each organized sector would therefore lobby for a positive own tariff and a non-positive tariff in all other sectors. However, when different sectors produce (imperfect) substitutes, the prices become strategic complements: a higher tariff (and price) in one sector implies a possibility for a higher price in the substitute-producing sector. Therefore in this situation the organized sectors would lobby for more protection in other sectors and, perhaps, somewhat less protection in its own sector. The argument for Bertrand competition in case domestic and foreign goods are non-homogeneous is very similar. As prices are strategic complements, an decrease in sector k tariff would cause a decrease in prices in all sectors $i = 1, ..., m$.. making lobby group's desired trade protection more moderate in case of substitutes.

4 The level of protection

Previous section has illustrated the mechanism through which demand linkages may lead to less protection for the organized industries and more for the non-organized ones. However, one potential criticism of this approach is that the analysis uses the "other things equal" assumption. That is, it bases the tariff comparison on exogenous variation of the right-hand-side variables, thereby ignoring the equilibrium feedback effect from tariffs to demand/profits etc.[8]

In this section we address this criticism by solving system (6) for the equilibrium trade tariffs. This

[8] Same criticism applies to the original PFS model.

allows us to explicitly evaluate the impact of substitutability on trade protection. To make the analysis tractable, we limit ourselves to the case of two non-numeraire goods, $m = 2$.

We focus on the following question: Assume that industry 1 is organized in a lobby group, but industry 2 is not. How do the trade tariffs in such an environment respond to the change in the degree of substitutability between the industries' products?

In answering it, we proceed in two steps. First, we assume that there is only one domestic and one foreign firm in each of the two markets. For this case we analytically prove that more product substitutability results in less protection. Next, we study how the relationship between protection and substitutability is affected by the number of firms in the industry. As this case proves difficult to characterize analytically, we provide the results of numerical simulations for different industry sizes.

Throughout this section the share of industry 1's owners in total population is denoted by α, and, as only industry 1 is organized, the total share of organized population $\alpha_L = \alpha$. For simplicity, we also assume equal productivity across sectors, $c_1 = c_2 = c$.

Case 3.1: Each sector has only one domestic and one foreign firm, $n_k = n_k^* = 1$**,** $k = 1, 2$**.**

In this case system (6) consists of two parametric equations linear in τ_1 and τ_2. Solving it, we obtain the equilibrium trade tariffs $\tau_k(\sigma, \alpha, a)$ in the organized industry $k = 1$, and the non-organized industry $k = 2$ as functions of the degree of substitutability, total size of the organized groups, weight of the social welfare in the government payoff function etc. The expressions themselves are technically involving and are thus relegated to the appendix.

Our aim is to show how the increase in the degree of substitution between the products affects the equilibrium trade protection. Specifically, we study how the trade tariffs in a lobbying equilibrium differ from the first-best trade policy benchmark $\tau_k^0(\sigma)$ and how does this difference change with σ.

However, the first-best trade tariffs $\tau_k^0(\sigma)$ are themselves declining in σ due to the strategic trade policy effect. The intuition is as follows: A domestic import tax in sector i is aimed at increasing the market share of the domestic firm. As the degree of substitutability increases, the trade tax in sector k also improves the market position of the firms in sector $-k$. Hence, the overall effect of sector k's tariff on the firm in sector k becomes weaker, leaving less room for strategic trade policy.

To control for it, we analyze the ratios of trade tariffs in industry k with and without lobbying

$$T_k(\sigma, \alpha, a) = \tau_k(\sigma, \alpha, a) / \tau_k^0(\sigma), \quad k = 1, 2, \tag{9}$$

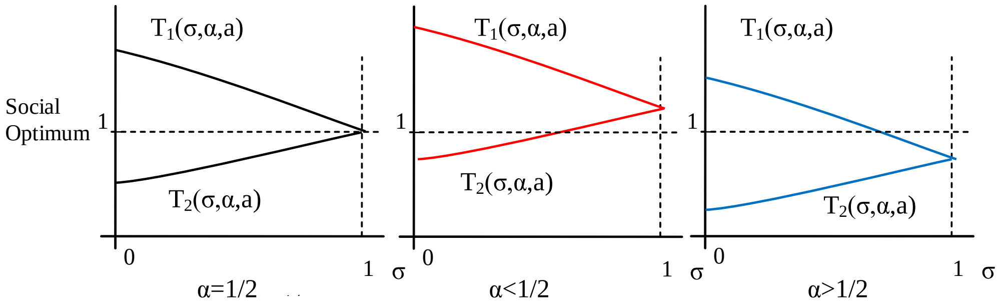
which we henceforth refer to as relative protection. We study the equilibrium response of relative protection rates to the change in the degree of substitution σ.

Proposition 1 *Consider an economy with two non-numeraire goods, where industry* 1 *is organized, while industry* 2 *is not. Then the equilibrium relative protection of the organized industry decreases with the degree of substitution, and the one of the non-organized industry increases with substitution:*

$$dT_1(\sigma,\alpha,a)\big/ d\sigma < 0, \quad dT_2(\sigma,\alpha,a)\big/ d\sigma > 0.$$

Thereby, Proposition 1 confirms the results of the previous section. Further, the trade tariffs imposed on the organized and non-organized industries converge as the threat of a demand shift becomes more and more serious. As $\sigma \to 1$, the goods become perfect substitutes, and industries 1 and 2 face a joint demand for these goods. Hence, the foreign firms in sector 1 and sector 2 become identical competitors from the point of view of the domestic organized industry 1, so it lobbies for the same tariff in both industries.Notice that it does not necessarily imply that the influence-driven protection rates converge to

Figure 1: Relative tariff as a function of the degree of substitutability.



the socially-optimal level as substitutability increases. For higher σ,the organized industry is interested in reducing the difference between the prices of its own good and the substitute, not in achieving the first-best outcome. For example, in case of perfect substitutes $\sigma = 1$ both industries face the same trade tariff. However, it may differ from from the first-best level in either direction depending on the ownership concentration of industry 1 (that is, of the population share of its owners α). If $\alpha = 1/2$, industry 1's owners are exactly representative of the entire population – they own as much of the firm's profit claims[9] as does the average person. Therefore, the lobbying equilibrium tariff matches the first-best tariff. If the ownership of industry 1 is more concentrated, $\alpha < 1/2$, it cares about the domestic profits, as opposed to consumer welfare, more than does the average person. In this case, the equilibrium protection rates exceed the socially optimal ones. Similarly, if $\alpha > 1/2$, the resulting equilibrium protection falls below the socially optimal level.

Still, if the degree of substitution is not too high, the organized industry always achieves more than the first-best protection. We saw earlier (equation (8)) that with zero substitutability the organized industry

[9] Note that in case of perfect substitutability, domestic firms in sectors 1 and 2 are identical as are their profit functions.

is always overprotected, $T_1(0, \alpha, a) > 1$. As the relative protection rate changes continuously, this also holds in some neighborhood of $\sigma = 0$. For the same reason, the non-organized industry is always underprotected as $T_2(0, \alpha, a) < 1$. These arguments are summarized in Corollary 2:

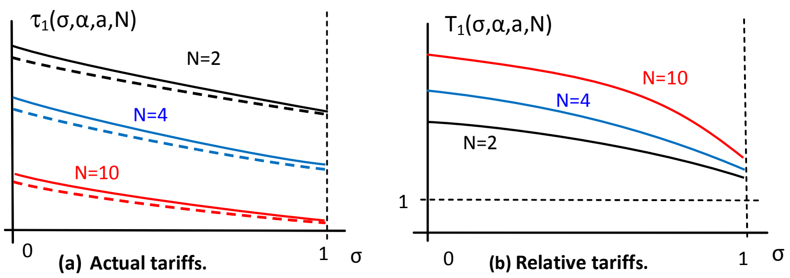
Corollary 2 *If the degree of substitutability is not too high, an increase in σ shifts the influence-driven protection rates towards the socially optimal levels.*

Figure 1 illustrates the findings of Proposition 1, Corollary 2 and the related discussion.

Case 3.2 Each sector has multiple domestic and foreign firms, $n_k = n_k^* > 1$, $k = 1, 2$.

Now we analyze how an increase in intra-industry competition affects the relationship between substitutability and equilibrium trade protection. For simplicity, we assume equal industry sizes across products: $N_1 = N_2 = N$, and that there are as many domestic as foreign firms in each industry. Then we compare the patterns of trade protection for different values of N. As above, we assume that only industry 1 is organized. As analytical results proved to be difficult, we resort to numerical solution. Figure 2

Figure 2: Actual and relative tariffs for different industry sizes.



Socially optimal tariffs/outcomes are shown by dashed lines

presents the results for industry 1's tariffs for $N = 2$, 4 and 10, both for the actual tariff $\tau_1(\sigma, a, \alpha, N)$ and for the relative tariff $T_1(\sigma, a, \alpha)$.

We see that the equilibrium protection rates decrease with the substitutability for different industry sizes., in line with the intuition discussed above. Also, not surprisingly, increased competition brings down both the lobbying equilibrium tariff $\tau_1(\sigma, N)$, and the respective first-best tariffs $\tau_0(\sigma, N)$.

Interestingly, the relative tariff T_1 increases in N, suggesting that the first-best tariff declines faster than the lobbying equilibrium tariff. However it is likely to be an artifact of our specification. In our setting total industry profits are more sensitive to the own import tariff as N increases, as the resulting cost advantage (vis-a-vis foreign firms) matters more when profit margins are tight. This, intra-market competition increases (relative) lobbying intensity. However, with different demand elasticity this result may no longer hold.[10]

The result of Corollary 2 and simulations in case 3.2.suggest a new explanation for a puzzle commonly observed in the empirical studies of PFS. Most of the studies report unexpectedly low estimates of the weight government puts on campaign contributions relative to social welfare. This implies that lobbies have little influence over the government decisions, and the government behaves almost as a social welfare maximizer. This finding has caused a concern about the empirical interpretation of the PFS model, as such an estimate would suggest that the protection is hardly "for sale".

This paper suggests that smaller deviations from the first-best trade policy may result from the weaker incentives to lobby, due to cross-product substitutability. Indeed, most of the studies cited above are based on 4-digit SIC industry data, which implies at least some degree of substitutability between the products of the different industries. By not accounting for the cross-product substitutability, the existing empirical analysis overestimates the organized industries' gains from protection. This may lead to a downward bias in the estimate of the relative weight the government puts on these gains (or, equivalently, resulting campaign contributions) when comparing them against social welfare losses from protection.

This argument implies that controlling for cross-product substitutability may improve the estimates of PFS. Notice that the above model only considers the case when all products are equally substitutable. However, it can be directly extended to a more realistic scenario with products being substitutable within (highly aggregated) sectors, and unrelated between these sectors. Then, other things equal, the model would predict less intense lobbying and lower level of protection in the (organized) subindustries of sectors with higher within-sector product substitutability. The 2009 working paper version of Bombardini and Trebbi (2012) suggest some evidence consistent with these predictions. They find that the lobbying expenditures are lower in 4-digit SIC industries producing more substitutable products. However, a more direct empirical verification of the substitutability effect is yet to be done.

5 Substitutability and lobbying activity

So far, as in the original PFS model, we assumed an exogenous lobby group structure. But why are some industries organized while other ones not? We argue that demand linkages may contribute to explaining this phenomenon. Indeed, in the previous section we demonstrated that if industry 1

[10]Recall that truthful equilibrium equilibrium concept requires interior solutions (and non-prohibitive tariffs). However, as competition increases, the government would be more willing to choose a prohibitive tariff in lobbying equilibrium (as it would benefits industry lobbies at little cost to consumer welfare). Thus, our model would be less applicable to there situation.
On the other hand, it is likely that the lobbying incentives in highly competitive situations will be weak anyway. For example, in case of perfect competition domestic profits would be zero independently of the own tariff. Further, a demand shift due to substitutability would still yield zero profits to the lobbying industry. So, in this (extreme) case intra-market competition completely eliminates lobbying incentives.

is organized, the import tax for the substitute product 2 increases with the degree of substitutability. Hence, a sufficiently high degree of substitutability may entail a situation where a non-organized industry becomes protected without paying for it. That may give rise to a free-riding behavior in lobbying. In this section we illustrate this reasoning with an example.

We focus on the incentives of an industry to get organized. To address this question, the existing game is extended by an initial stage 0 of costless lobby formation. In this stage, industries simultaneously decide whether they get organized. The resulting lobbies play our standard lobbying game in stage 1.

To clarify the argument we return to the 2-sector-2 firms of case 3.1, but impose some additional assumptions. First, the non-numeraire goods are perfect substitutes, $\sigma = 1$. Second, the sectors have the same size $\alpha_1 = \alpha_2 = \alpha$, and allow some of the voters to own no claims to any of the industries' profit, so $\alpha \leq 1/2$. We also assume that in the lobbying stage the organized industries use globally truthful strategies, that is, that the contributions of lobby j are (globally) equal to the excess of the lobby's welfare over a certain threshold B_j. All truthful equilibria are also locally truthful, so we can rely on the results obtained in previous sections.

We proceed as follows: first, we characterize the outcome of the (original) tariff-setting game under three possible situations - when both, only one or none of the industries got organized in stage zero. Then we turn back to the lobby forming stage and address industry's incentives to get organized.

As discussed in section 4, the trade tariffs on both goods are the same due to perfect substitutability between the goods. Denote the equilibrium with no lobbies by $\mathbf{\Omega}^0$ and the respective tariff(s) by τ^0, the equilibrium and the tariff with a single organized industry by $\mathbf{\Omega}^s$ and τ^s, and the (symmetric)[11] equilibrium and the tariff with both firms organized by $\mathbf{\Omega}^b$ and τ^b.

Lemma 3 *In 2-sector-2 firms model with perfect substitutes the equilibrium protection rates increase with lobby participation:*

$$\tau^b \geq \tau^s \geq \tau^0 > 0.$$

In absence of any lobbying the government chooses socially optimal protection rate. If a single industry is organized, the tariff increases towards the industry-preferred one. However, in this case the non-organized industry gains more from resulting trade protection than the organized one. Indeed, they face the same trade tariffs, but only the organized industry pays for it. When both industries actively

[11] In the original PFS setting, the interests of different industries are opposed to each other. So, if organized, each industry indeed chooses to buy protection. However, in the presence of substitutes, this outcome is not necessarily unique. If both industries are allowed to lobby, but each of them can get protected without paying for it, the game may admit equilibria where only one of two organized industries is active. Alternatively, there can be equilibria where both industries lobby but make different contributions. We study symmetric truthful equilibria, which seems to be natural given the symmetry of the setting.Their existence is proved in 2011 working paper version of this paper.

participate in lobbying, the resulting policy is even more biased towards the interest groups' preferred import tax. However, now both lobbies contribute to the government, and the total lobby contribution is higher to compensate for the social welfare loss. The following lemma describes how the industries' net-of-contribution welfare compares between the equilibria.

Lemma 4 *a) Both industries' welfare in no-lobbying equilibrium $\Omega^0$is lower than in any of the lobbying equilibria Ω^s, Ω^b;*

b) Industry lobbying both in Ω^s and in Ω^b, has a higher net welfare in Ω^b than in Ω^s;

c) Industry lobbying only in Ω^b has a lower net welfare in Ω^b than in Ω^s if and only if its size $\alpha > 1/7$.

As long as the share of population that owns the industry is sufficiently large, it loses from participating in lobbying. The intuition behind this result is as follows: if the industries are very concentrated ($\alpha \leq 1/7$), they highly benefit from an increase in protection as the loss in their members' consumer welfare resulting from higher prices is negligible. With decreasing ownership concentration ($\alpha > 1/7$), more and more consumers in the industry lose from the price increase which results in a decrease in protection rates and industry welfare. And it may be the case that the gain of the industry from increased protection (due to this industry participation in lobbying) is not high enough to cover the necessary contribution for this increase.

Now, we proceed backwards to the lobby formation stage. In this stage, each industry decides whether it gets organized (O) and buys influence in the next stage of the game, or stays non-organized (N) and remains passive in the lobbying game. The original PFS setup entails a single equilibrium of a type (O,O): getting organized is a dominant strategy in that game as different lobbies' interests are strictly opposed to each other. The same happens in our setting if the industry's ownership structure is very concentrated ($\alpha < 1/7$). Then, again, getting organized is a dominant strategy and a single (O,O) equilibrium emerges.

However, if an industry has a dispersed ownership structure (higher α), it prefers to commit not to lobby as long as the substitute industry will be lobbying. That is, the lobby formation game has two pure strategy Nash equilibria, (O,N) and (N,O), and one mixed strategy equilibrium. The outcome (O,O) is no longer an equilibrium of the game. So, in the presence of substitutability, fewer industries get organized, and lobbying becomes less intense. Finally, by continuity the results of this section also extend to markets with high, but not perfect substitutability (σ close to 1).This discussion is summarized in the following proposition.

Proposition 5 *Consider a 2-sector-2 firms lobbying game, extended by the participation decision stage.*

If cross-product demand linkages are sufficiently strong, a dispersed ownership structure weakens the industries' incentives to get organized which leads to less intensive lobbying.

This proposition can be given an alternative interpretation: if an industry is sufficiently concerned about its members' consumer welfare, it is less interested in getting organized. This is in line with the result in Bombardini and Trebbi (2009), who show that sectors with higher labor-to-capital ratio are characterized by lower levels of lobbying expenditures.

One natural question to ask is whether this less intensive lobbying behavior is efficient from the industries' *joint* point of view. The answer is no. The total gain in industries' welfare resulting from the joint participation in lobbying is not fully offset by increasing cost of lobbying.

Corollary 6 *The aggregate net welfare of industries* 1 *and* 2 *is larger in equilibrium* $\mathbf{\Omega}^b$ *when they both participate in lobbying than in equilibrium* $\mathbf{\Omega}^s$ *when only one of the lobbies is active.*

To sum up, this section illustrates the mechanism through which demand linkages can caused the free-riding problem in lobbying: The product substitutability produces a positive inter-industry externality from protection which, in turn, may lead to dilution of the incentives to organize and less protection than would be jointly optimal for the industries.

Again, this result is in line with the findings of 2009 working paper version of Bombardini and Trebbi (2012), who document less lobbying in industries producing more substitutable products. However, more empirical tests are needed to check whether the effect is due to free-riding in lobbying.

6 Conclusion

This paper studies the impact of the demand linkages on the political rivalry among the interest groups in the Grossman and Helpman (1994) "Protection for Sale" framework. It is known that the political competition in the original PFS model is purely due to the interest groups concerned with the well-being of their members as consumers. This paper introduces product substitutability and oligopolistic competition into the PFS framework to provide a more compelling justification for the political rivalry among different lobbies. It analyses the determination of trade policy and the intensity of inter-industry lobbying competition in the presence of these demand linkages.

The paper shows that the product substitutability weakens organized interest groups' incentives to lobby. The threat of losing demand to the substitute product makes organized industries' lobbying strategy less aggressive, which results in lower protection rates. Therefore, by not accounting for product substitutability, the original model overstates the organized groups' desire for protection. This result may explain why the empirical investigations of the PFS model have found that the government is predominantly concerned with welfare rather than contributions by lobbies. The paper then turns to the endogenous lobby formation. It demonstrates that product substitutability also weakens industries' incentives to get organized due to the possibility to free-ride on the substitute-producing industry's lobbying effort.

The suggested framework can be extended in a number of directions. In particular, one can use it to analyze strategic interactions between the domestic and foreign government. Alternatively one can study how demand linkages may affect the government's choice of policy instruments, such as import tariffs vs. production subsidies. Another important question that is left behind in the paper is the empirical verification of the suggested model. These extensions are part of the future research agenda.

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A Appendix

A.1 Outputs, profit and imports sensitivity to trade tariffs

Lemma A.1 *a) In Cournot-Nash equilibrium of the production stage* $\partial q_k(\tau)/\partial\tau_k > 0$, $\partial q_k^*(\tau)/\partial\tau_k < 0$, $\partial q_k(\tau)/\partial\tau_j \geq 0$ *and* $\partial q_k^*(\tau)/\partial\tau_j \geq 0$ *for* $j \neq k$, *and* $\partial\pi_k(\tau)/\partial\tau_j > 0$.
b) Matrix $\left[\left(\partial m_k(\tau)/\partial\tau_j\right)_{k,j}\right]$ *is invertible and* $H = -\left[\left(\partial m_k(\tau)/\partial\tau_j\right)_{k,j}\right]^{-1} \geqq 0$.

Proof: FOCs of profit maximization for each the domestic/foreign firms in sector k yield

$$1 - n_k q_k - n_k^* q_k^* - \sigma \sum_{s\neq k}\left(n_s q_s + n_s^* q_s^*\right) = c_k + q_k, \quad k = 1, ..., m \tag{10}$$

$$1 - n_k q_k - n_k^* q_k^* - \sigma \sum_{s\neq k}\left(n_s q_s + n_s^* q_s^*\right) = c_k + q_k^* + \tau_k, \quad k = 1, ..., m. \tag{11}$$

It follows that $q_k = q_k^* + \tau_k$, $\forall k$. So the systems (10), (11) can be rewritten in a matrix form as

$$\mathbf{Rq} = \mathbf{1} - \mathbf{c} + \mathbf{n}^*\tau \quad \text{and} \quad \mathbf{Rq}^* = \mathbf{1} - \mathbf{c} - \mathbf{n}\tau,$$

where the respective matrices are given by

$$\mathbf{R} = \begin{pmatrix} 1+\mathrm{N}_1 & \sigma\mathrm{N}_2 & ... & \sigma\mathrm{N}_m \\ \sigma\mathrm{N}_1 & 1+\mathrm{N}_2 & ... & \sigma\mathrm{N}_m \\ ... & ... & ... & ... \\ \sigma\mathrm{N}_1 & \sigma\mathrm{N}_2 & ... & 1+\mathrm{N}_m \end{pmatrix}, \mathbf{n}^* = \begin{pmatrix} \mathrm{n}_1^* & \sigma\mathrm{n}_2^* & ... & \sigma\mathrm{n}_m^* \\ \sigma\mathrm{n}_1^* & \mathrm{n}_2^* & ... & \sigma\mathrm{n}_m^* \\ ... & ... & ... & ... \\ \sigma\mathrm{n}_1^* & \sigma\mathrm{n}_2^* & ... & \mathrm{n}_m^* \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 1+\mathrm{n}_1 & \sigma\mathrm{n}_2 & ... & \sigma\mathrm{n}_m \\ \sigma\mathrm{n}_1 & 1+\mathrm{n}_2 & ... & \sigma\mathrm{n}_m \\ ... & ... & ... & ... \\ \sigma\mathrm{n}_1 & \sigma\mathrm{n}_2 & ... & 1+\mathrm{n}_m \end{pmatrix}.$$

Matrix R is invertible as its determinant is positive

$$\det R = (1 + N_1) \prod_{i=2}^{m} (1 + (1 - \sigma) N_i) + \sigma \sum_{i=2}^{m} N_i \prod_{j=1, j \neq i}^{m} \left(1 + (1 - \sigma) N_j\right) > 0.$$

Therefore, $\partial q_k / \partial \tau_j$ are given by the respective elements of the matrix $S = R^{-1} n^*$, and $\partial q_k^* / \partial \tau_j$ - by the elements of $S^* = -R^{-1} n$. The diagonal elements of S are positive. E.g. for $k > 1$

$$S_{kk} = \frac{1}{\det R} \left(\left((1 + N_1) \prod_{i=2, i \neq k}^{m} (1 + (1 - \sigma) N_i) + \sum_{i=2, i \neq k}^{m} \sigma N_i \prod_{j=1, j \neq i, k}^{m} \left(1 + (1 - \sigma) N_j\right) \right) n_k^* \right.$$
$$\left. - \sum_{i=1, i \neq k}^{m} \left(\sigma N_i \prod_{j=1, j \neq i, k}^{m} \left(1 + (1 - \sigma) N_j\right) \right) \sigma n_k^* \right) = \frac{n_k^* \left[(1 + N_1) - \sigma^2 N_1\right]}{\det R} \prod_{i=2, i \neq k}^{m} (1 + (1 - \sigma) N_i) > 0$$

Similarly, one shows that $S_{kj} \geq 0$ and $S_{kj}^* \geq 0$ for all $k \neq j$, and that $S_{kk}^* < 0$ for all k, which proves the results for the output sensitivity to tariffs. Finally, $\pi_k(\tau) = q_k^2$, which yields the last result in part (a).

To prove (b) notice that by definition of imports $m_k(\tau)$ and the derivations above

$$\left[\left(\partial m_k(\tau) / \partial \tau_j \right)_{k,j} \right] = diag\left(n_1^*, n_2^*, ..., n_m^*\right) \left[\left(\partial q_k^*(\tau) / \partial \tau_j \right)_{k,j} \right] = -diag\left(n_1^*, n_2^*, ..., n_m^*\right) R^{-1} n,$$

where $diag\left(n_1^*, n_2^*, ..., n_m^*\right)$ denotes a diagonal matrix. Matrices n and R have identical structure, so by the same argument as above $\det n > 0$, which proves invertibility. Further,

$$H = n^{-1} R \; diag\left(\frac{1}{n_1^*}, \frac{1}{n_2^*}, ..., \frac{1}{n_m^*} \right).$$

Similarly to the proof in (a), the diagonal elements of $n^{-1} R$ are positive, and the off-diagonal elements nonnegative, which completes the proof of of Lemma A.1.

A.2 Solution of system (6) in case $m = 2$ and $n_k = n_k^* = 1$, $k = 1, 2$

The solution of system (6) with respect to the trade tariffs yields

$$\tau_1(\sigma, \alpha, a) = (1 - c) \frac{4(a+1)(a+2\alpha)\sigma^3 + 4(\alpha^2 - 3a\alpha - 2a - 3a^2 - 3\alpha)\sigma^2 - (9a^2 + 26a\alpha + 10a + 13\alpha^2 + 14\alpha)\sigma + (3a + \alpha + 2)(9a + 11\alpha)}{4(a + 2\alpha - 1)(a + 2\alpha)\sigma^4 + (14a - 118a\alpha - 45a^2 - 81\alpha^2 + 22\alpha)\sigma^2 + (9a + 11\alpha - 2)(9a + 11\alpha)}$$
$$\tau_2(\sigma, \alpha, a) = (1 - c) \frac{4a(a + 2\alpha - 1)\sigma^3 - 4(a + 2\alpha + 3a\alpha - \alpha^2 + 3a^2)\sigma^2 + (18a - 26a\alpha - 9a^2 - 13\alpha^2 + 14\alpha)\sigma + (3a + \alpha)(9a + 11\alpha - 2)}{4(a + 2\alpha - 1)(a + 2\alpha)\sigma^4 + (14a - 118a\alpha - 45a^2 - 81\alpha^2 + 22\alpha)\sigma^2 + (9a + 11\alpha - 2)(9a + 11\alpha)}. \tag{12}$$

In turn, system (6) with $I_1 = I_2 = \alpha = 0$ yields the first-best trade tariffs $\tau_i^0(\sigma) = (1 - c) / (\sigma + 3)$.

A.3 Proof of Proposition 1

To keep the analysis tractable, we only provide the proof in case $\alpha = 0$ (which as argued in section 2 is the case where the lobbying incentives of the industry are most illustrative). The proof for a general value of $\alpha \in [0, 1]$ can be found in the 2011 working paper version of this paper.

To show that $\partial T_1(\sigma, 0, a) / \partial \sigma < 0$ for $\sigma \in 0, 1$ and any admittable a, we proceed in 4 steps.

1. Describe the necessary condition on the tariff to achieve an interior solution.

2. Represent $T_1(\sigma, 0, a)$ as a ratio of two polynomials $T_1(\sigma, 0, a) = \frac{N(\sigma,a)}{D(\sigma,a)}$. Show that $\partial T_1(\sigma, \alpha, a)/\partial\sigma < 0$ on [0, 1] iff $N_\sigma(\sigma, a)/D_\sigma(\sigma, a) > T_1(\sigma, 0, a)$ for all $\sigma \in [0, 1]$ and a delivering an interior solution.

3. Introduce an auxiliary function $A(\sigma, a)$ and show that $N_\sigma(\sigma, a)/D_\sigma(\sigma, a) \geq A(\sigma, a)$ for any admissible (σ, a).

4. Show that $A(\sigma, a) \geq T_1(\sigma, a)$ for any admissible (σ, a).

Steps 2, 3 and 4 imply that $\partial T_1(\sigma, 0, a)/\partial\sigma < 0$. The proof $\partial T_2(\sigma, 0, a)/\partial\sigma > 0$ is similar.

Step 1. Necessary condition on the tariff for the interior solution.

We concentrate on non-prohibitive tariffs (i.e., positive imports). Using $\tau_1(0, 0, a)$ from system (12) in industry 1's import at $\sigma = 0$ we obtain the necessary condition for the interior solution

$$a \geq 2. \tag{13}$$

Step 2. Necessary and sufficient condition for $T_1(\sigma, 0, a)$ to decline in σ.

System (12) and definition (9) give the following expression for $T_1(\sigma, 0, a)$

$$T_1(\sigma, 0, a) = \frac{4\sigma^4(a+1) - 4\sigma^3(2a+1) - 3\sigma^2(7a+6) + 2\sigma(9a+4) + 9(3a+2)}{4\sigma^4(a-1) - \sigma^2(45a-14) + 81a - 18}$$

Denote the numerator of $T_1(\sigma, 0, a)$ by $N(\sigma, a)$ and the denominator by $D(\sigma, a)$.

$D(\sigma, a)$ decreases in σ for any admissible a. Indeed, using condition (13) we have

$$\partial D(\sigma, a)/\partial\sigma = 2\sigma(8\sigma^2(a-1) - 45a + 14) \leq 2(6 - 37a) < 0$$

This also means that $D(\sigma, a)$ is positive for $\sigma \in [0, 1]$, as $D(1, 0, a) = 8a(5a-1) > 0$.

As $\partial D(\sigma, a)/\partial\sigma < 0$ and $D(\sigma, a) > 0$,

$$\frac{\partial T_1(\sigma, a)}{\partial\sigma} < 0 \quad \Leftrightarrow \quad \frac{N_\sigma(\sigma, a)}{D_\sigma(\sigma, a)} > \frac{N(\sigma, a)}{D(\sigma, a)} \equiv T_1(\sigma, 0, a).$$

Step 3. Auxiliary function $A(\sigma, a)$, such that $\frac{N_\sigma(\sigma,a)}{D_\sigma(\sigma,a)} \geq A(\sigma, a)$.

Define $A(\sigma, a)$ - a linear function of σ

$$A(\sigma, a) = 3\frac{a+2}{9a-2}\left(1 - \frac{8}{(37a-6)}\sigma\right).$$

Consider

$$\Delta_1 = \frac{N_\sigma(\sigma, a)}{D_\sigma(\sigma, a)} - A(a, 0, \sigma) = \frac{4(\sigma-1)}{(9a-2)(37a-6)}\frac{L(\sigma, a)}{D_\sigma(\sigma, a)} \tag{14}$$

where $L(\sigma, \alpha, a)$ is a cubic polynomial with respect to σ,

$$\begin{aligned} L(\sigma, \alpha, a) &= 3a(37a-6)(9a-2) + \sigma(37a-6)(2a+63a^2) \\ &\quad + 8\sigma^2(a+2\alpha)(21a+10)(9a-2) + 32\sigma^3 a(a-1)(a+2). \end{aligned}$$

To sign Δ_1, note that $D_\sigma(\sigma, a) \leq 0$ and $\sigma - 1 < 0$. Further, $(9a-2) > 0$ and $(37a-6) > 0$ by (13) so the coefficients of the polynomial $L(\sigma, \alpha, a)$ are also positive. Thus, for any $\sigma \in 0, 1$, $\Delta_1 \geq 0$.

Step 4. Proof of $A(\sigma, a) \geq T_1(\sigma, 0, a)$.

Consider the difference Δ_2 between $T_1(\sigma, \alpha, a)$ and $A(\sigma, a)$:

$$\Delta_2 = T_1(\sigma, a) - A(\sigma, a) = 4\sigma M(\sigma, a) / [(9a-2)(37a-6) D(\sigma, a)] \tag{15}$$

where $M(\sigma, a)$ is a 4th degree polynomial with respect to σ,

$$M(\sigma, a) = 3a(9a-2)(18-31a) - 2\sigma a(37a-6)(2+9a) + \sigma^2 a(243a^2 - 280a + 68) + 2\sigma^3 a(37a-6)(5a+2) + 8\sigma^4 a(a-1)(a+2).$$

To sign Δ_2, notice that, as shown above, $(9a-2)(37a-6) D(\sigma, a) > 0$. Consider $M(\sigma, a)$.

$$\partial^3 M(\sigma, a) / \partial\sigma^3 = 12a((37a-6)(5a+2) + 16\sigma(a-1)(a+2)) > 0.$$

Thus, $\partial^2 M(\sigma, a) / \partial\sigma^2$ increases at $\sigma \in 0, 1$, and

$$\partial^2 M(\sigma, a) / \partial\sigma^2 > \partial^2 M(0, a) / \partial\sigma^2 = 2a(243a^2 - 280a + 68) > 2a(243a(a-2) + 68) > 0.$$

So, $M(\sigma, a)$ is convex in σ at 0, 1, and reaches its maximum at (either of) the segment's corners. But $M(\sigma, a) < 0$ both at $\sigma = 0$ and $\sigma = 1$. Therefore $M(\sigma, a) \leq 0$, which implies $\Delta_2 \leq 0$.

A.4 Proof of Lemma 3

In absence of lobbying the tariff is socially optimal, $\tau^0 = (1-c)/4$. The expressions for lobby equilibria tariffs

$$\tau^s = \frac{(1-c)}{4} \frac{5a + \alpha + 2}{(5a + 7\alpha - 1)} \quad \text{and} \quad \tau^b = \frac{(1-c)}{4} \frac{5a + 2\alpha + 4}{5a + 14\alpha - 2}.$$

are obtained by solving system (6) for $I_1 = 1$, $I_2 = 1$, and $\alpha_L = \alpha$, and $I_1 = I_2 = 1$, and $\alpha_L = 2\alpha$, respectively (with $m = 2$, $n_k = n_k^* = \sigma = 1$). Lemma's result then follows from the general from of condition (13), $a + 3\alpha \geq 2$.

A.5 Proof of Lemma 4

By definition, in a truthful equilibrium, lobby j chooses the anchor B_j to make government indifferent between the equilibrium policy τ and the policy τ^{-j}, chosen if the contributions of lobby j were zero. Then, the contribution $C_j(\tau, B_j)$ of lobby j solves

$$\sum_{i \in L, i \neq j} C_i(\tau_{-j}, B_i) + aW(\tau^{-j}) = \sum_{i \in L} C_i(\tau, B_i) + aW(\tau). \tag{17}$$

Consider equilibrium $\mathbf{\Omega}^s$ where only industry 1 is organized. If it were to contribute zero, the government would maximize social welfare

$$\tau^s_{-j} = \arg\max_{\tau \in \mathbf{T}} aW(\tau) = \tau_0. \tag{18}$$

From (17) and (18), it follows that the contribution of lobby 1 is

$$C_1\left(\tau^s, B_1^s\right) = aW\left(\tau^0\right) - aW(\tau^s).$$

The net welfare of lobby 1 is

$$V_1(\tau^s) = W_1(\tau^s) - C_1(\tau^s, B_1^s) = W_1(\tau^s) - aW(\tau^0) + aW(\tau^s). \tag{19}$$

As industry 2 is not organized, its net welfare is equal to its gross welfare, $V_2(\tau^s) = W_2(\tau^s)$**.**

Now, turn to $\mathbf{\Omega}^b$. Industry 1 does not make any contribution at τ^b_{-1}

$$C_1(\tau^b_{-1}, B_1) = \max\left[0,\, W_1(\tau^b_{-1}) - B_1\right] = 0. \tag{20}$$

As $\sigma = 1$, industries 1 and 2 are exactly alike, and the equilibrium $\mathbf{\Omega}^b$ is symmetric, $W_1(\tau^b_{-1}) = W_2(\tau^b_{-1})$ and $B_1 = B_2$. Therefore, industry 2 does not contribute anything at the tariff vector τ^b_{-1} either. Hence

$$\tau^b_{-1} = \arg\max_{\tau \in \mathbf{T}} aW(\tau) = \tau^0. \tag{21}$$

Conditions (17), (20) and (21) and symmetry imply

$$C_1(\tau^b, B_1^b) = C_2(\tau^b, B_2^b) = 1/2\left(aW(\tau^0) - aW(\tau^b)\right).$$

The net payoff of either lobby group is thus

$$V_i(\tau^b) \equiv B_i^b = W_i(\tau^b) - C_i(\tau^{\mathbf{b}}, B_i^b) = W_i(\tau^b) - 1/2\left(aW(\tau^0) - aW(\tau^b)\right), \quad i = 1, 2. \tag{22}$$

Substituting the expressions for the outputs, profits, etc. into the expression for the lobby's welfare (4) and aggregate social welfare, and simplifying yields

$$V_1(\tau^b) - V_1(\tau^s) = \frac{9}{20} \frac{(1-c)^2 a (1-2\alpha)^2}{(5a + 7\alpha - 1)(5a + 14\alpha - 2)} \geq 0. \tag{23}$$

$$V_2(\tau^b) - V_2(\tau^s) = \frac{9}{20} \frac{(1-c)^2 a (1-2\alpha)^2}{(5a + 14\alpha - 2)(5a + 7\alpha - 1)^2} (1 - 7\alpha) \quad \begin{cases} > 0, \alpha < 1/7; \\ \leq 0, \alpha \geq 1/7. \end{cases} \tag{24}$$

Finally, the lobby group welfare is the lowest in no-lobbying equilibrium $\mathbf{\Omega}^0$, as lobbies can always offer zero contributions in $\mathbf{\Omega}^b$ and $\mathbf{\Omega}^s$.

A.6 Proof of Corollary 6

Follows from (23) and (24).